

Cobordism, $ER = EPR$, and the Sum Over Topologies

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Based on:

- 1909.10355 and 2004.06738 with Cumrun Vafa
- Work in Progress



The Missing Corner

Despite its successes, string theory remains an incomplete theory. This **missing corner** means that we cannot directly ask string theory how the mysteries of quantum gravity are resolved. (Brennan, Carta, Vafa '17)

The Swampland Program, with its focus on **physical principles**, serves as a useful lens for peering into the missing corner. In this talk, I will focus on one of these mysteries:

**In a UV complete theory of quantum gravity,
are we supposed to sum over topologies?**

Ex: Quantum foam for top. A-model. (Iqbal, Nekrasov, Okounkov, Vafa '03)

Goal: Advocate for a specific answer on the basis of Swampland principles.

$$Z_{\text{grav}}(\mathbb{C}^3) = \sum \text{[diagram of a vertex with three edges and a small triangle]}$$

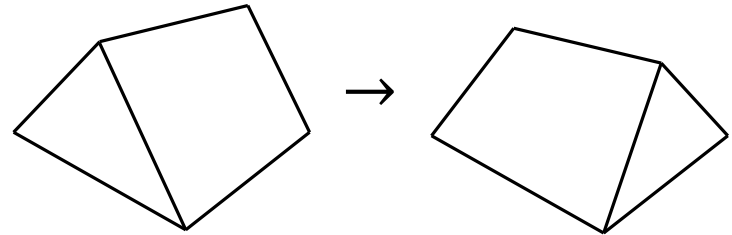
Outline

In this talk, I will:

1. Quickly review classic arguments for and against a sum over topologies.
2. Introduce the main thesis: the sum over topologies can be **entirely removed via gauge fixing** in any UV complete QG.
3. Explain how holography defines a gauge redundancy of QG (ER = EPR).
4. Organize the sum over topologies into a sum over **cobordism classes** and apply the cobordism conjecture.

Summing Topologies is Obviously Right

Gravity is defined by a **dynamical spacetime manifold**; quantum mechanics instructs us to sum all dynamically allowed processes. Dynamical topology change is allowed in string theory!



Cannot restrict the sum over global topologies by any local rule, so any restriction on topology would violate the **equivalence principle**.

Summing topologies frequently gives the right answer. **Ex:** BH entropy from Euclidean Schwarzschild.

More recently: replica wormholes reproduce Page curve (Penington, Shenker, Stanford, Yang '20) and quantify violation of global symmetries. (Chen, Lin '20, Hsin, Iliesiu, Yang '20, Bah, Chen, Maldacena '22)

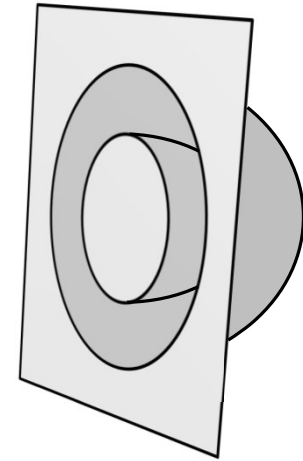
Summing Topologies is Obviously Wrong

The inclusion of nontrivial topologies in the path integral leads to a **loss of unitarity**. Clear for Euclidean wormholes/baby universes, (Coleman '88, Giddings, Strominger '88, '89) but also plays a role in the **BH information paradox**.

Relatedly, cannot put Lorentzian metric on manifold with topology change. See (Harlow, Shaghoulian '20).

Most fatally, seems incompatible with holography due to the **factorization paradox**:

$$Z(\bigcirc \bigcirc) = \bigcirc \bigcirc + \bigcirc \bigcirc + \dots \neq Z(\bigcirc)^2.$$



A Radical Proposal

Motivated by the factorization paradox, ('t Hooft, '93) made the following radical proposal: “...the functional integral describing black holes probably has to be limited to topologically trivial field configurations only.”

Let us evaluate this proposal:

- It is deeply flawed as part of a definition of QG for the reasons described previously.
- It avoids all the paradoxes of a sum over topologies!

Small Modification: 't Hooft's proposal is correct but arises as the result of **gauge fixing** rather than as part of the definition of the theory. See (Eberhardt, '21).

Main Result: Assuming $ER = EPR$ and the cobordism conjecture, such a gauge fixing is possible in any UV complete QG.

ER = EPR as a Gauge Redundancy

To make my argument, we will need to understand gauge redundancies of QG that relate different topologies (so the bulk topology is not gauge-invariant).

The existence of such gauge redundancies follows from the combination of dynamical topology change with the **Hamiltonian constraint**: bulk dynamics in QG are pure gauge. (Jafferis, '17)

A schematic proposal for such gauge redundancies is given by the **ER = EPR** conjecture: (Maldacena, Susskind '13)

$$| \text{wormhole} \rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n/2} | \text{loop } n \rangle \otimes | \bar{n} \rangle.$$

ER = EPR is a gauge redundancy that relates one configuration to a quantum superposition of other configurations.

ER = EPR on Histories

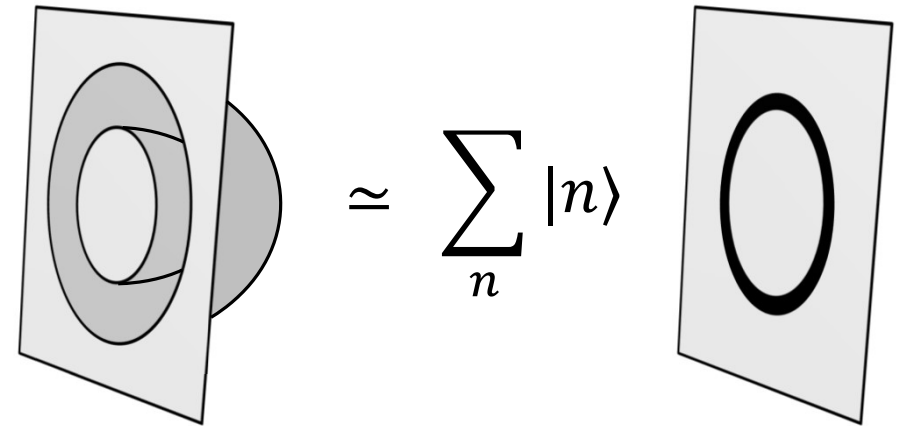
Any gauge redundancy on states also defines a gauge redundancy on histories.

Let us apply ER = EPR to the Euclidean spacetime describing a virtual BH loop.

In the “ER gauge,” we have a smooth spacetime manifold of **nontrivial topology**.

In the “EPR gauge,” we have a coherent sum over BH microstates running in loops on top of a **topologically trivial** spacetime.

We have successfully replaced a contribution to the path integral from a nontrivial topology with a sum of **stringy excitations** on the trivial topology.

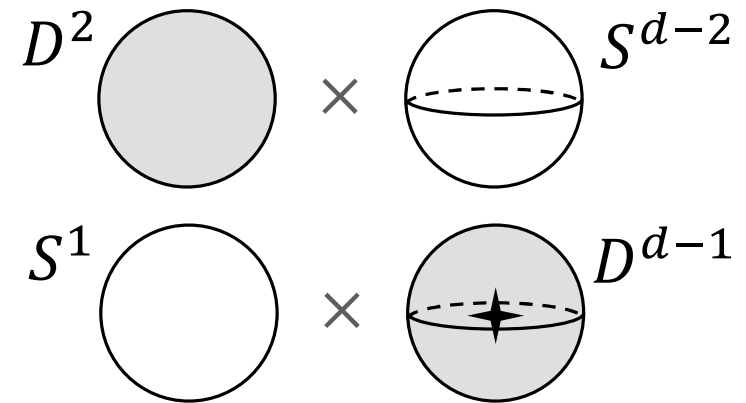


Surgery via ER = EPR

To proceed to more general topologies, we need to understand how exactly the topology changes when we apply ER = EPR.

In the “ER” gauge, a neighborhood of the black holes is given by Euclidean Schwarzschild, with topology $D^2 \times S^{d-2}$:

In the “EPR” gauge, a neighborhood of the microstates has topology $S^1 \times D^{d-1}$, with the microstates wrapping $S^1 \times \text{pt}$:

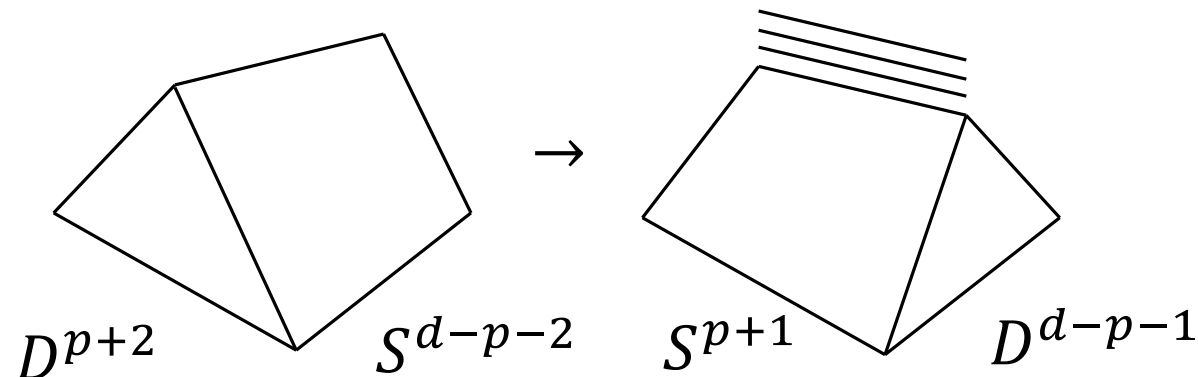


Our spacetime topology has changed by a **surgery**.

More generally, surgery allows us to replace an open set of topology $D^{p+2} \times S^{d-p-2}$ with $S^{p+1} \times D^{d-p-1}$ by cutting and gluing along their common boundary. To perform such a surgery on spacetime, we interpret $D^{p+2} \times S^{d-p-2}$ as a Euclidean black p -brane and apply “**ER = EPR for p -branes**.”

Surgery and Geometric Transition

The operation of surgery is extremely familiar in string theory: it is simply **geometric transition**, which is the basic geometry of holography as realized in string theory.



ER = EPR can be thus be understood as an example of geometric transition. The first manifold is **equivalent** to a specific coherent superpositions of wrapped branes on the second.

This picture is realized in stringy realizations of ER = EPR (Jafferis, Schneider '21) as well as in the topological vertex. (Aganagic, Klemm, Marino, Vafa '03)

What Does Surgery Get Us?

Consider the gravitational partition function:

$$\mathcal{Z} = \sum_M \mathcal{Z}_M, \quad \mathcal{Z}_M = \int_M \mathcal{D}\phi \, e^{-S(\phi)}.$$

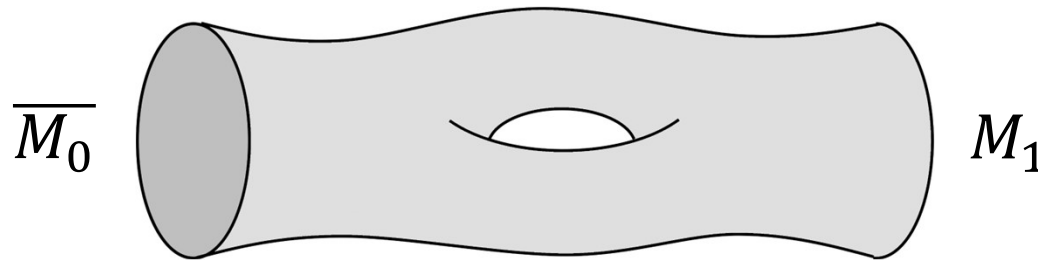
Suppose two manifolds M_0, M_1 are related by a sequence of surgeries. By applying a sequence of ER = EPR gauge transformations, we can replace \mathcal{Z}_{M_0} with some path integral over M_1 involving a **modified sum over stringy degrees of freedom**, or vice versa.

Collect all manifolds into **surgery classes**, collections of manifolds related by some sequence of surgeries. We can gauge fix the sum over topologies into a restricted sum including only one representative of each surgery class.

Thus, assuming ER = EPR, we can reorganize the sum over topologies into a sum over surgery classes.

Surgery and Cobordism

When are two manifolds M_0 , M_1 related by a sequence of surgeries? A standard result in differential topology tells us two manifolds are related by surgery if and only if they are **cobordant**. Thus, surgery classes are the same thing as cobordism classes.



Given a sequence of surgeries, we can construct a cobordism known as the **trace** of the surgery (the process traced out by performing the surgeries in sequence).

Given a cobordism, we can choose a **Morse function**. Then the manifold changes by a surgery every time we cross a critical point.

The Swampland Cobordism Conjecture

In a UV complete theory of quantum gravity, we expect that **all cobordism classes should be trivial**. (JM, Vafa '19)

The argument was based on the absence of global symmetries and the expectation of string universality.

Now, we see a different motivation for the cobordism conjecture: it describes the **irreducible sectors in the sum over topologies** after taking $ER = EPR$ into account.

Assuming $ER = EPR$, the cobordism conjecture states that UV complete QG must admit a description without any sum over topologies. See (JM, Vafa '20).

Such a description makes manifest that the paradoxes associated with a sum over topologies can be avoided in UV complete QG.

Conclusion

I have argued that UV complete theories of quantum gravity admit **ER = EPR gauge transformations that change the spacetime topology**.

I identified cobordism as quantifying the irreducible sectors in the sum over topology and argued that **we may absorb the sum over topologies into a modified sum over stringy degrees of freedom on any fixed topology**.

There are (at least) a few points where this argument could be made sharper:

- How does ER = EPR work for extended objects, such as p-branes?
- How does surgery work for manifolds with singularities (cobordism defects)?
- What if \exists black hole microstates with nontrivial topology? Solve recursively?
- Most importantly: how can we check this picture in any toy model?

Thank You For Listening!

Bonus: The Role of Cobordism Defects

The cobordism conjecture predicts the existence of singularities known as **cobordism defects**, which are required to perform more generalized types of surgeries that result in manifolds with singularities or connect manifolds in different cobordism classes.

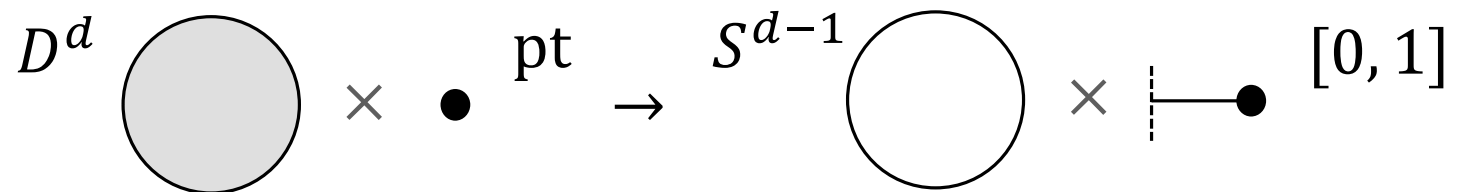
For example, suppose we want to apply ER = EPR to a black $(d - 4)$ -brane with horizon topology \mathbb{RP}^2 . Since $[\mathbb{RP}^2]$ is nontrivial, the microstates must include a cobordism defect. The resulting generalized surgery takes the following form:

$$D^{d-2} \times \mathbb{RP}^2 \rightarrow S^{d-3} \times D^3/\mathbb{Z}_2$$

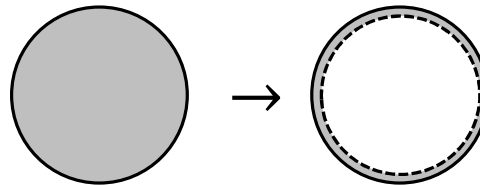
The singularity D^3/\mathbb{Z}_2 is a cobordism defect, realized in Type IIA as an O6-plane.

End of the World Branes

There is one universal cobordism defect in every theory: the **end-of-the-world brane**, required to trivialize the cobordism class $[\text{pt}]$ of a single point. The associated generalized surgery takes the following form:

$$D^d \times \text{pt} \rightarrow S^{d-1} \times [0, 1]$$


Taking the products, we obtain:



Some superposition of end-of-the-world branes should lead to a type of **subregion holography**: a particular choice of gauge where a subregion has been replaced by degrees of freedom living on its boundary.