

A dS from higher dimensions

G. Bruno De Luca - Stanford University

Based on

2104.13380 with Silverstein and Torroba

2104.12773 with Tomasiello

2109.11560, 2212.02511, 2306.05456 with De Ponti, Mondino, Tomasiello

+ work in progress

Swamplandia, Madrid

Sep 15, 2023

Two challenging problems

- Top down prescribes extra dimensions

$$S = m_D^{D-2} \int \sqrt{-g_D} R_D + \dots$$

1. How to describe 4-dimensional physics? [separation of scales]

- Swampland conjectures
- Proposed compactifications
- For AdS, CFT constraints

[Ooguri, Vafa, '07,
Lüst, Palti, Vafa, '19, ...]

[Kachru, Kallosh, Linde, Trivedi '03, DeWolfe, Giryates, Kachru,
Taylor, '05,
Polchinski, Silverstein '09,
Petrini, Solard, Van Riet '13,
Cribiori, Junghans, Van Hemelryck, Van Riet, Wrase '21, ...,
Carrasco, Coudarchet, Marchesano, Prieto, '23]

[Polchinski, Silverstein '09,
Conlon, Quevedo '18, Alday, Perlmutter '19,
Apers, Montero, Van Riet, Wrase '22, ...]

2. How to describe *realistic* 4-dimensional physics? [dS or more general acc. expansion]

- Conjectures, explicit constructions, consistency conditions, ...

- **This talk:** how to get constraints from the equations of motion and a way to evade them

Physics of gravity compactifications

- At low energies $S = m_D^{D-2} \int \sqrt{-g_D} R_D + \text{matter}$

$$ds_D^2 = e^{\frac{2}{D-2}f(y)}(g_d^\Lambda(x) + g_n(y))$$

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- At low energies $S = m_D^{D-2} \int \sqrt{-g_D} R_D + \text{matter}$

- Equations of motion

$$\frac{1}{D-2} e^{-f} \Delta(e^f) = \frac{1}{d} \hat{T}^{(d)} - \Lambda$$

$$R_{mn} - \nabla_m \nabla_n f + \frac{1}{D-2} \nabla_m f \nabla_n f = \Lambda g_{mn} + \tilde{T}_{mn}$$

- Spectrum of spin 2 fluctuations given by

$$\Delta_f \psi_i \equiv \Delta \psi_i - \nabla f \cdot \nabla \psi_i = m_i^2 \psi_i$$

- What can we prove in general, that applies to any solution?

$$ds_D^2 = e^{\frac{2}{D-2} f(y)} (g_d^\Lambda(x) + g_n(y))$$

$$\left[\hat{T}^{(d)} \equiv m_D^{2-D} g_d^{\mu\nu} \left(T_{\mu\nu} - \frac{T}{D-2} g_{\mu\nu} \right), \tilde{T}_{mn} \equiv m_D^{2-D} \left(T_{mn} - \frac{T^{(d)}}{d} g_{mn} \right) \right]$$

$$g_{4\,\mu\nu}(x) = g_{\mu\nu}^{(\Lambda)}(x) + \sum_i h_{\mu\nu}^i(x) \psi_i(y)$$

[Csaki, Erich, Hollowood, Shirman, '00,
Bachas, Estes, '11]

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smooth internal space, no boundaries:

$$\frac{1}{D-2} e^{-f} \Delta(e^f) = \frac{1}{d} \hat{T}^{(d)} - \Lambda \quad \Longrightarrow \quad d\Lambda = \int_{M_n} \sqrt{g_n} e^f \hat{T}^{(d)} \leq 0 \text{ for classical sources and no O-planes}$$

[Gibbons '84, de Wit, Smit, Hari Dass '87, Maldacena-Nuñez, '00]

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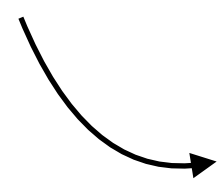
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- Synthetic Ricci curvature in effective dimension $N = 2 - d$
- Studied in the Optimal Transport literature, controls the spectrum of Δ_f

[Sturm '06, Lott, Villani '07, Villani '09, Ambrosio, Gigli, Savaré 14, ...]

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- Useful to prove [theorems on the spectrum](#) of $\Delta_f \equiv \Delta - \nabla f \cdot \nabla$ and bound $m_{KK}^2 / |\Lambda|$

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[GBDL, Tomasiello, '21]

For fluxes, scalar fields, scalar potentials, D-dim cosmological constants and localized sources with positive tension

Reduced Energy Condition

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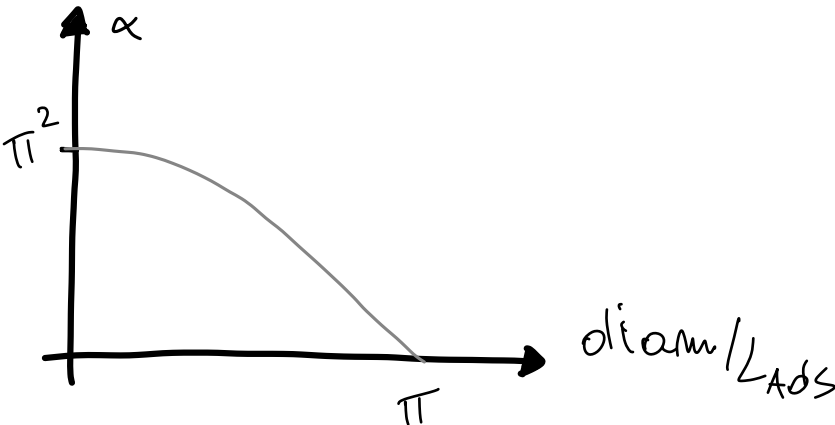
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For fluxes, scalar fields, scalar potentials, D-dim cosmological constants and localized sources with positive tension

- If $m_0 = 0 \implies \psi_0 = \text{const.}$ Reduced Energy Condition [GBDL, De Ponti, Mondino, Tomasiello, '23]

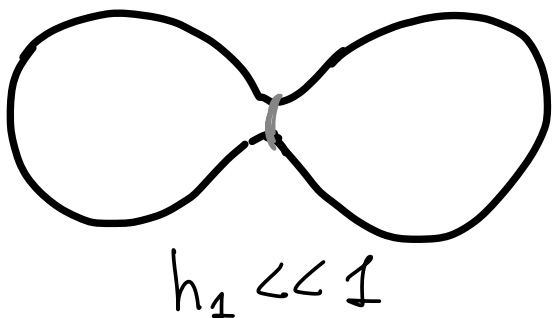
[GBDL, De Ponti, Mondino, Tomasiello, '22]

$$\frac{m_1^2}{|\Lambda|} \geq \alpha(\text{diam}/L_{AdS}) \frac{L_{AdS}^2}{\text{diam}^2}$$



Separation of scales achieved if $\text{diam} \ll L_{AdS}$
 Intuitive, but now rigorous even with D-brane singularities and warping

$$m_1^2 \geq \frac{1}{4} h_1^2$$



Cheeger constant [Cheeger '69]



Rigorous even in presence of O-planes
 Can be used to check sep. of scale in explicit proposed examples. e.g. in

[DeWolfe, Giryavets, Kachru, Taylor, '05 Acharya, Benini, Valandro '06, Junghans '20, Marchesano, Palti, Quirant, Tomasiello '20]

$$h_1^2 \sim N^{-1/2}, |\Lambda| \sim N^{-3/2}$$

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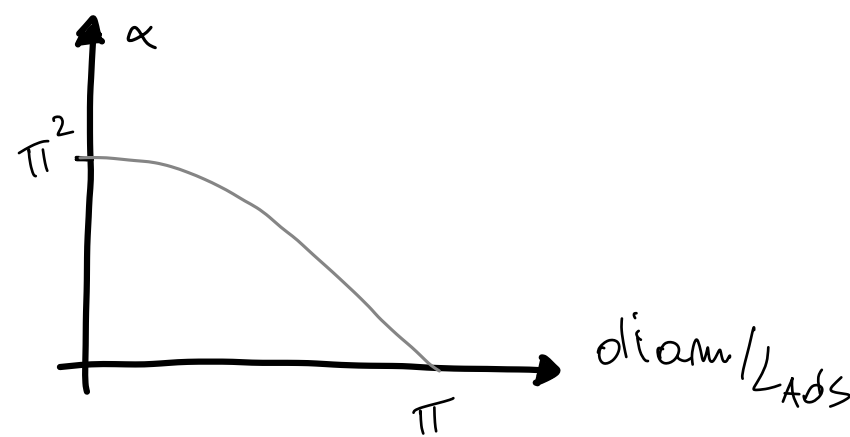
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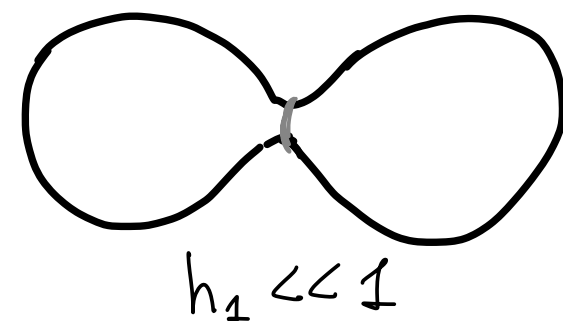
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$$h_1^2 \sim N^{-1/2}, |\Lambda| \sim N^{-3/2}$$

- Also upper bounds, e.g:

$$m_k^2 \leq a(n) \max \left\{ \sup(\partial f)^2, \frac{1}{n-1} \left(|\Lambda| + \frac{1}{D-2} \sup(\partial f)^2 \right) \right\} + b(n) k^{2/n} \text{Vol}_f^{-2/n}$$

[GBDL, Tomasiello, '21 using Hassannezhad, '13]

- Even assuming the REC, these do not exclude separation of scales

[cf. Collins, Jafferis, Vafa, Xu, Yau, '22,
Cribiori, Junghans, Van Hemelryck, Van Riet, Wrase '21]

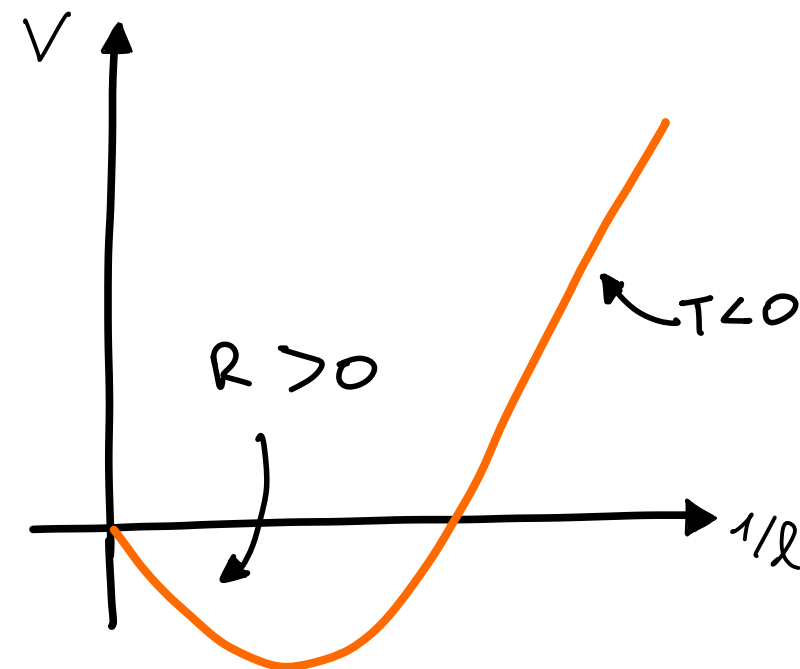
Violating the REC

- Generically, with only “positive energy” ($T^{(d)} < 0$), it is easy to stabilize positive internal curvature
 - A simple understanding is through the effective potential
 - Equivalent to the D-dimensional eoms after the warp-factor constraint is enforced

[Douglas, '09]

$$V_{\text{eff}}[g_n, \phi; u] = m_D^{2-D} \int_{M_n} \sqrt{g_n} u^2 \left(-R_n - 3 \frac{(\nabla u)^2}{u^2} - T_\phi^{(d)} \right)$$

$$ds_D^2 = u(y) ds_4^2(x) + ds_n^2(y)$$



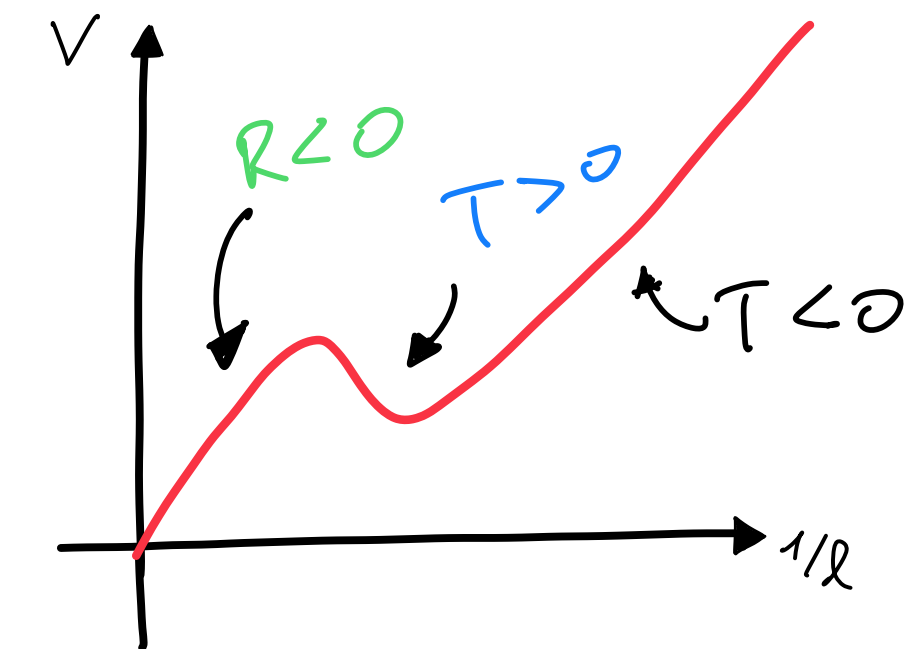
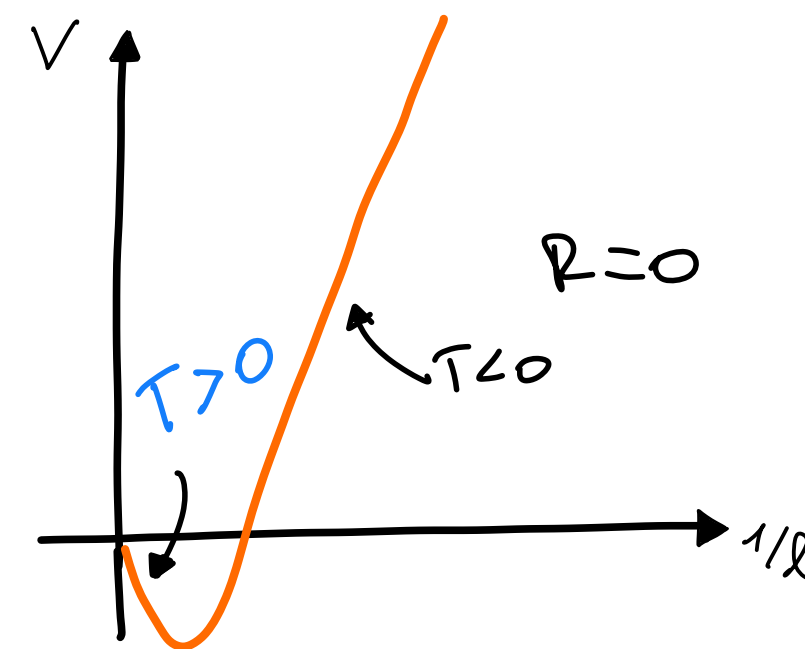
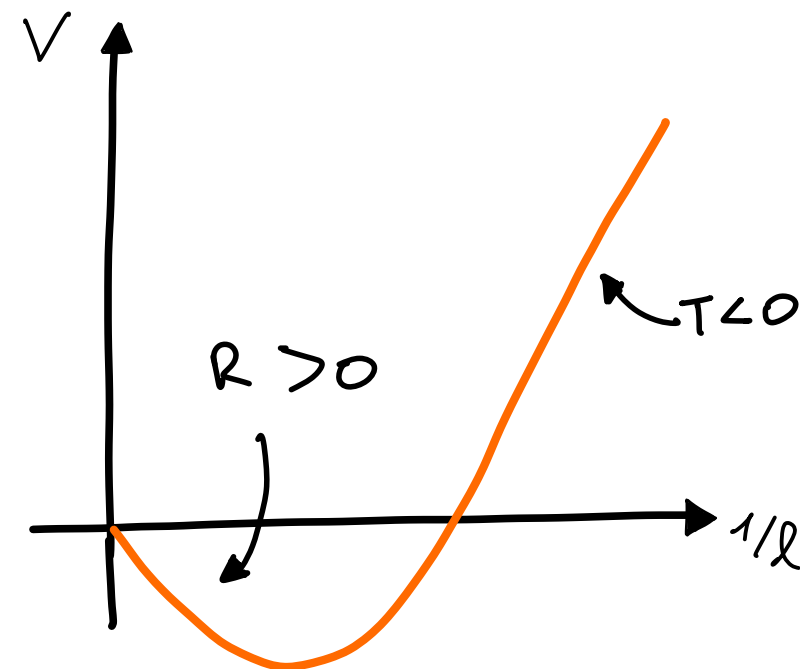
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- But if $T_\phi^{(d)}$ includes also negative contributions, one can stabilize zero and negative curvature [cf. Douglas, Kallosh, '10]
 - Much richer structure: the length (diameter) and KK modes are not tied to the curvature
 - Negative curvature in particular has no moduli (rigidity)

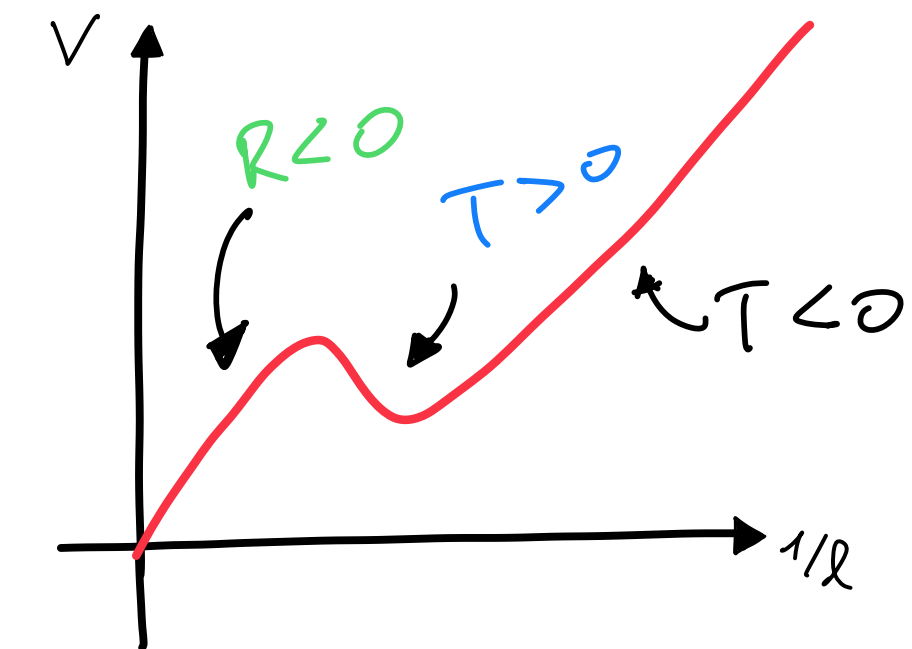
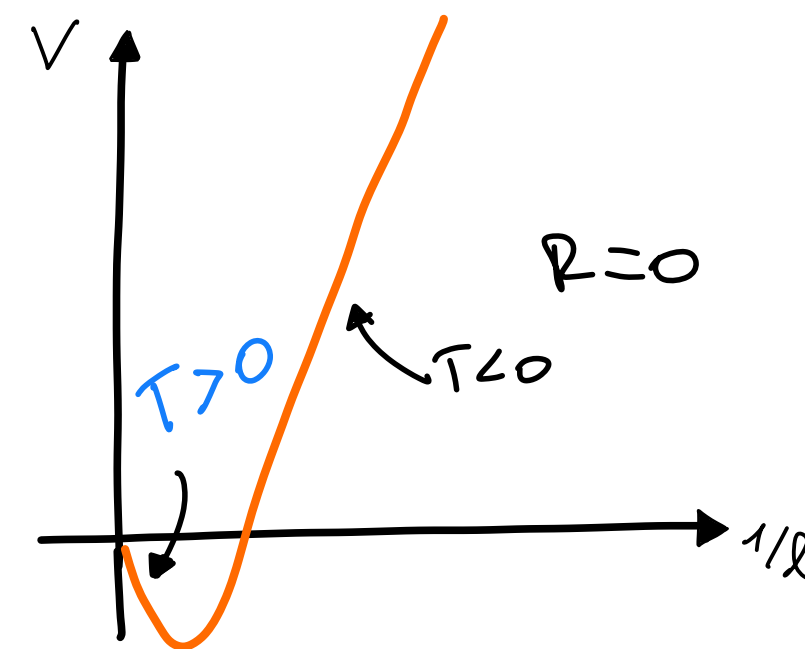
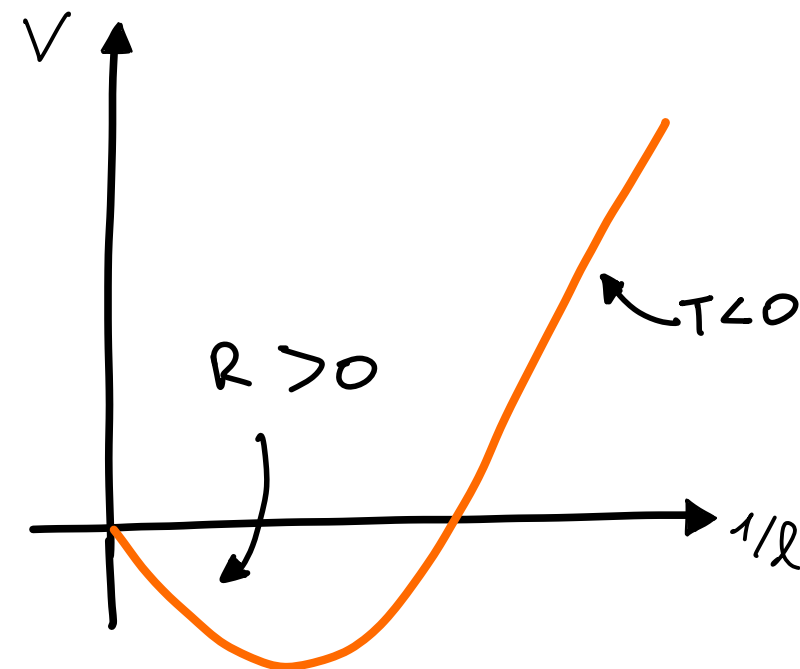
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 - Much richer structure: the length (diameter) and KK modes are not tied to the curvature
 - Negative curvature in particular has no moduli (rigidity)
- Many other possibilities for negative energy and uplift, (KKLT, LVS, supercritical,...)
 - Another simple possibility: O-planes and large gradients

[Silverstein, Torroba, Dodelson, Dong '13;
Córdova, GBDL, Tomasiello, '18 '19]

$$R_n = 0 + \text{Casimir} \rightarrow \Lambda_4 < 0$$

[GBDL, De Ponti, Mondino, Tomasiello, '22]

- With a compact internal space, **Casimir energy** density can be **automatically generated**
- If the space has **small circles**, with antiperiodic BCs for fermions, Casimir energies are of the form

$$T_{ij} \sim R_c(y)^{-D} g_{ij} \quad T_{ab} \sim -\frac{D-k}{k} R_c(y)^{-D} g_{ab}$$

other directions circle directions small circle size

[Arkani-Hamed, Dubovsky, Nicolis, Villadoro '07]

[cf. Maldacena, Milekhin, Popov '18]

- Then solve the **semi-classical equations**:
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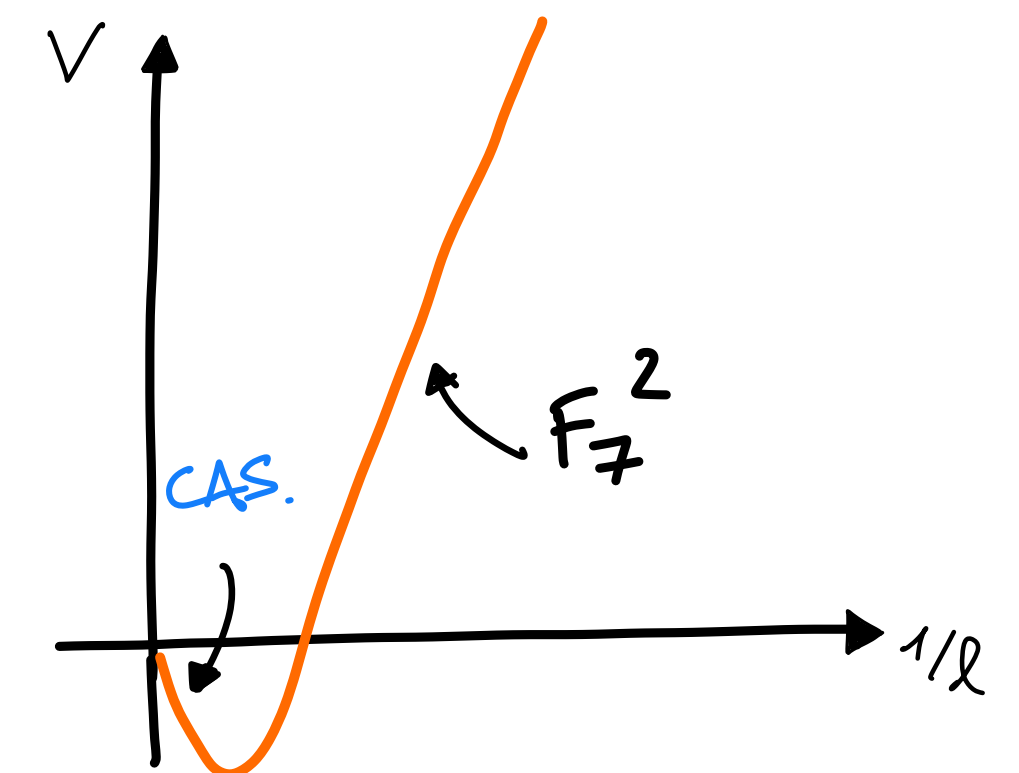
- Explicitly in **M-theory** on $\text{AdS}_4 \times T^7$:

$$ds_{11}^2 = L_4^2 ds_{\text{AdS}_4}^2 + R_c^2 ds_{T^7}^2$$

$$T_{\mu\nu}^{\text{Cas}} = |\rho_c| \ell_{11}^9 R_c^{-11} g_{\mu\nu} \quad T_{ij}^{\text{Cas}} = -\frac{4}{7} |\rho_c| \ell_{11}^9 R_c^{-11} g_{ij}$$

$$F_7 = f_7 \text{vol}_{T^7} \quad \frac{1}{\ell_{11}^6} \int F_7 = N_7$$

$$\Rightarrow \frac{L_4^2}{R_c^2} = \frac{2401}{4608} \frac{N_7^6}{\rho_c^4} \gg 1$$



$$\frac{R_c^{11}}{\ell_{11}^{11}} \sim N_7^{22/3} \gg 1$$

QG effects under control

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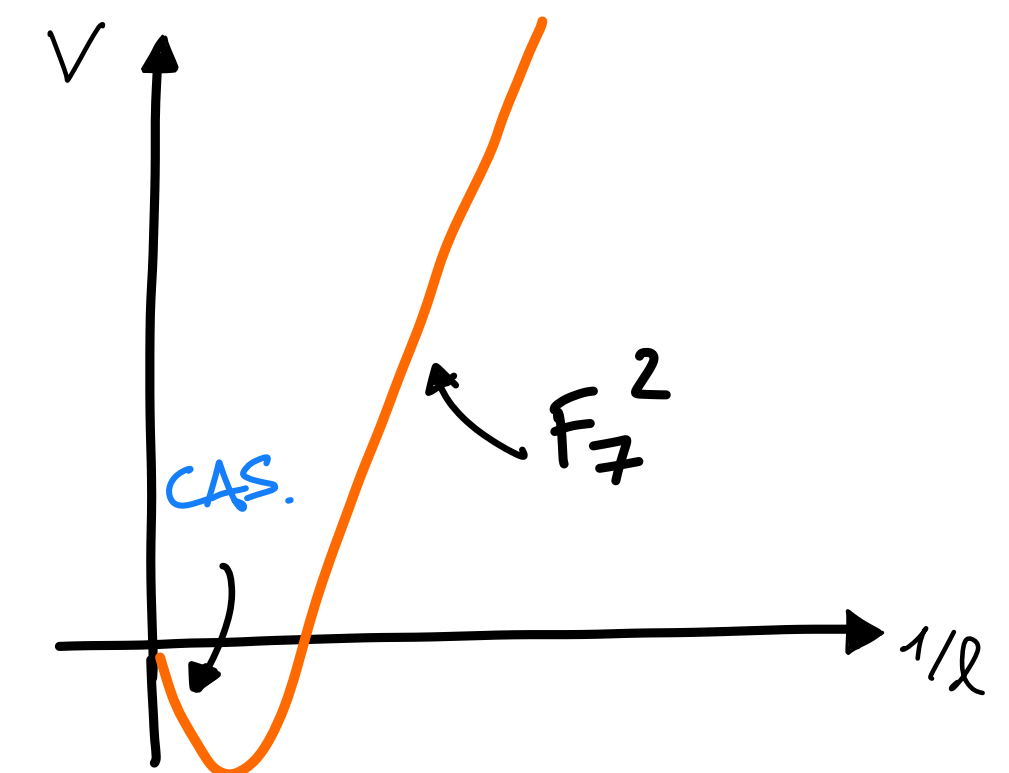
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QG effects under control

parametric separation of scales!

- Non-susy and unstable for M2 bubble nucleation
- Compatible with AdS distance conjecture, $m_{KK}^2 \sim |\Lambda|^{1/d}$

[Lust, Palti, Vafa, '19
Gonzalo, Ibáñez, Valenzuela, '21]



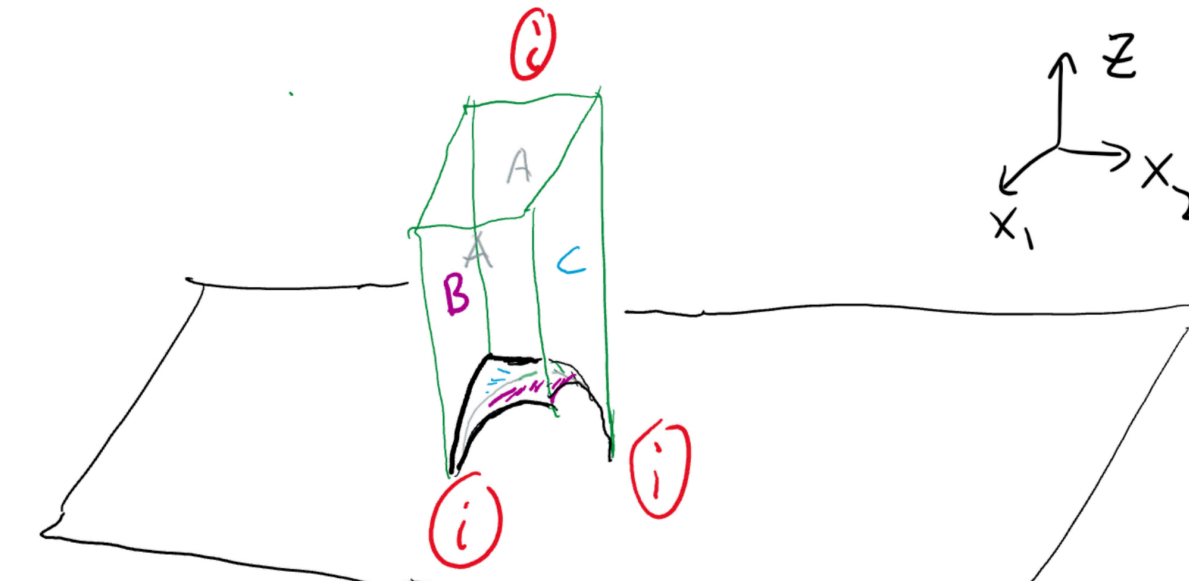
- [Also non stable dS possible in this way but not under parametric control]

Hyperbolic manifolds $\rightarrow \Lambda_4 > 0$

[e.g. Vinberg '93, Ratcliffe '06]

- **Negative curvature** and **explicit metric**, smooth manifolds. Quotients of hyperbolic space by subgroups Γ of its isometries
 - Recent explicit constructions by gluing right-angled polytopes

[Italiano, Martelli, Migliorini , '20]

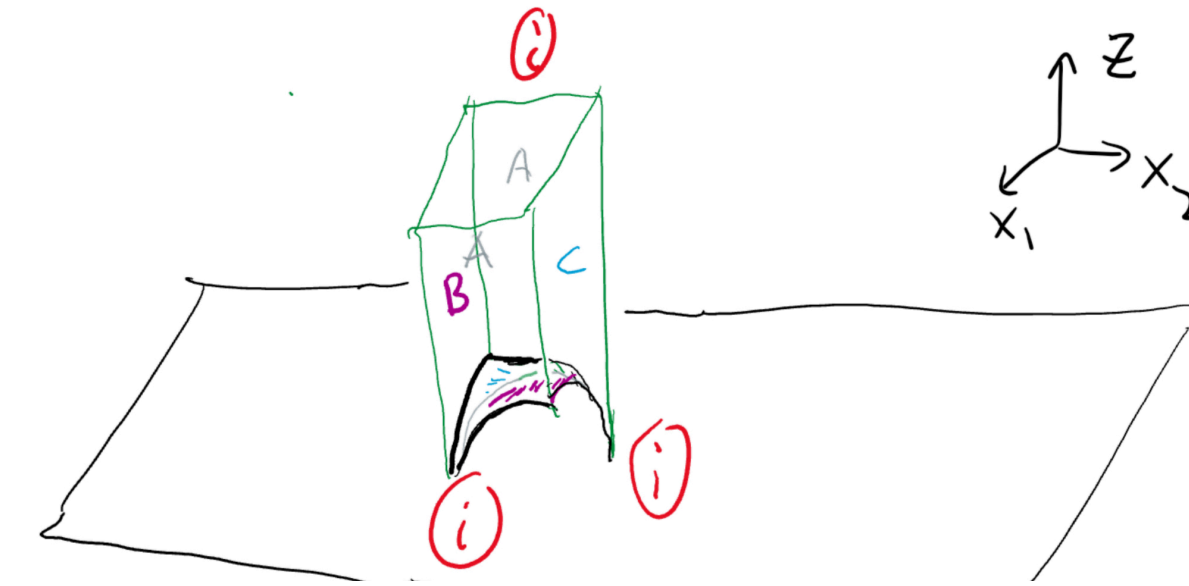


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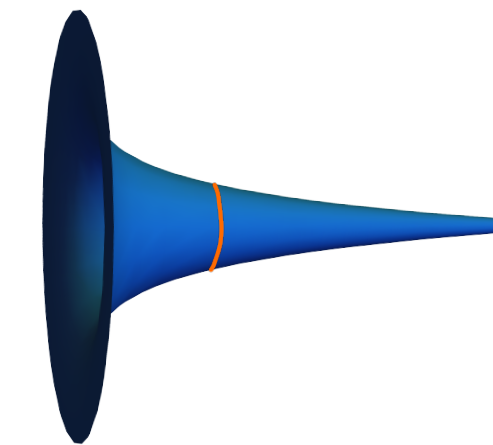
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[Italiano, Martelli, Migliorini , '20]



- Have one or more **cusps**: regions with **small slowly varying circles**

$$ds_{\mathbb{H}_7/\Gamma}^2 = dy^2 + e^{-\frac{2y}{\ell_7}} ds_{T^6}^2 \quad R_c \quad 0 \leq y \leq y_c$$



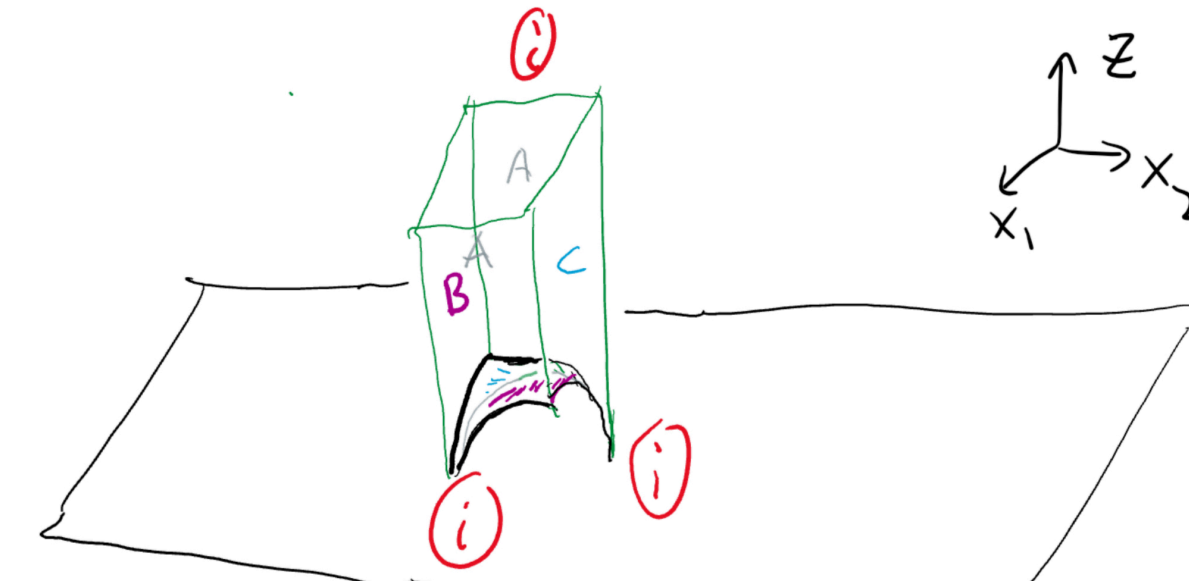
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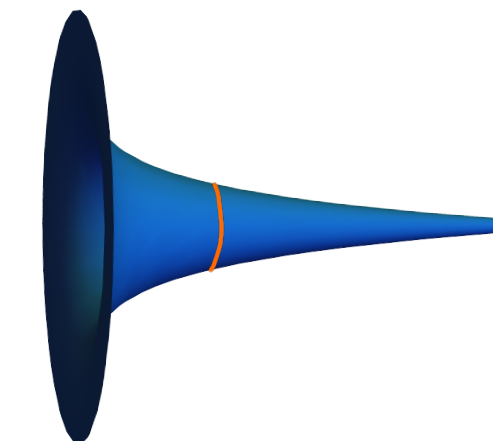
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- Cusps can be capped off in a **smooth** way: Anderson-Dehn filling to **compact** Einstein spaces

[Anderson '06]

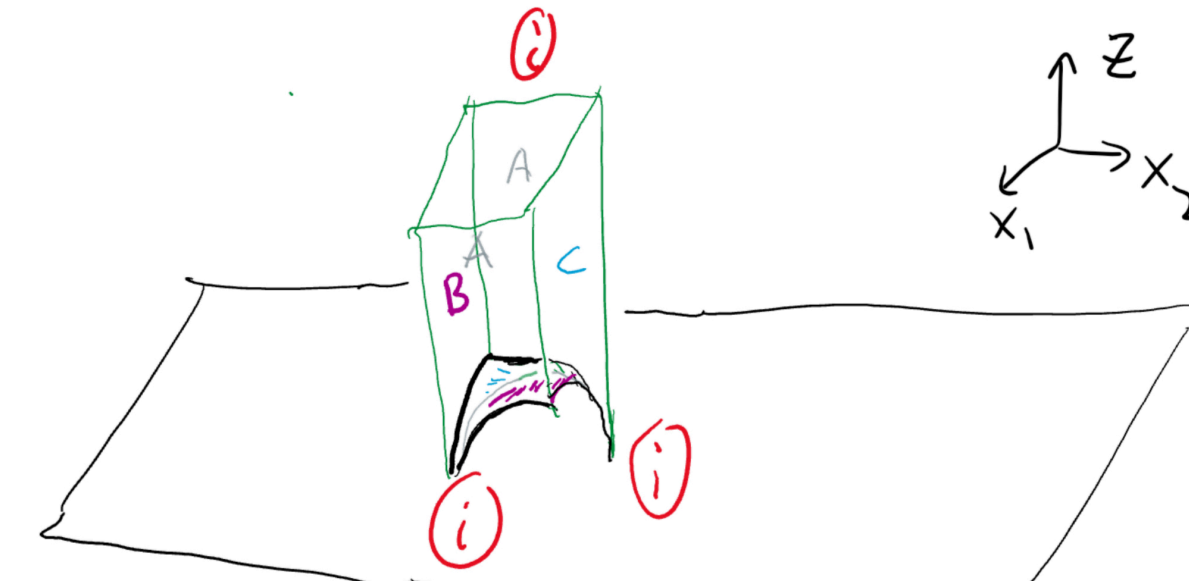
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[Anderson '06]

- They are **rigid** in $d > 2$: the hyperbolic structure is completely determined by the topology (no moduli space)

[Anderson '06]

- Also the filled Einstein manifold are rigid

- $-R_7$ is gapped at second order in h_{ij}

[e.g. Besse '87]

- Rigidity: essentially, we only have to stabilize the volume modulus [cf. Kaloper, March-Russell, Starkman, Trodden, '00]

$$ds_7^2 = \ell_7^2 \hat{ds}_7^2$$

$$V_{\text{eff}}[g_7, C_6] = \frac{1}{2\ell_{11}^9} \int_{M_7} \sqrt{g} u^2 \left(\underbrace{-R_7 - 3 \frac{(\nabla u)^2}{u^2}}_{\alpha \sim \ell_7^{-2}} \underbrace{- \ell_{11}^9 \rho_c R_c(y)^{-11}}_{\beta \sim \ell_7^{-11}} + \underbrace{\frac{1}{2} |F_7|^2}_{\gamma \sim \ell_7^{-14}} \right)$$

- Important: the negative contribution sits in the middle!

$$\alpha \sim \ell_7^{-2} \quad \beta \sim \ell_7^{-11} \quad \gamma \sim \ell_7^{-14}$$

- Rigidity: essentially, we only have to stabilize the volume modulus [cf. Kaloper, March-Russell, Starkman, Trodden, '00]

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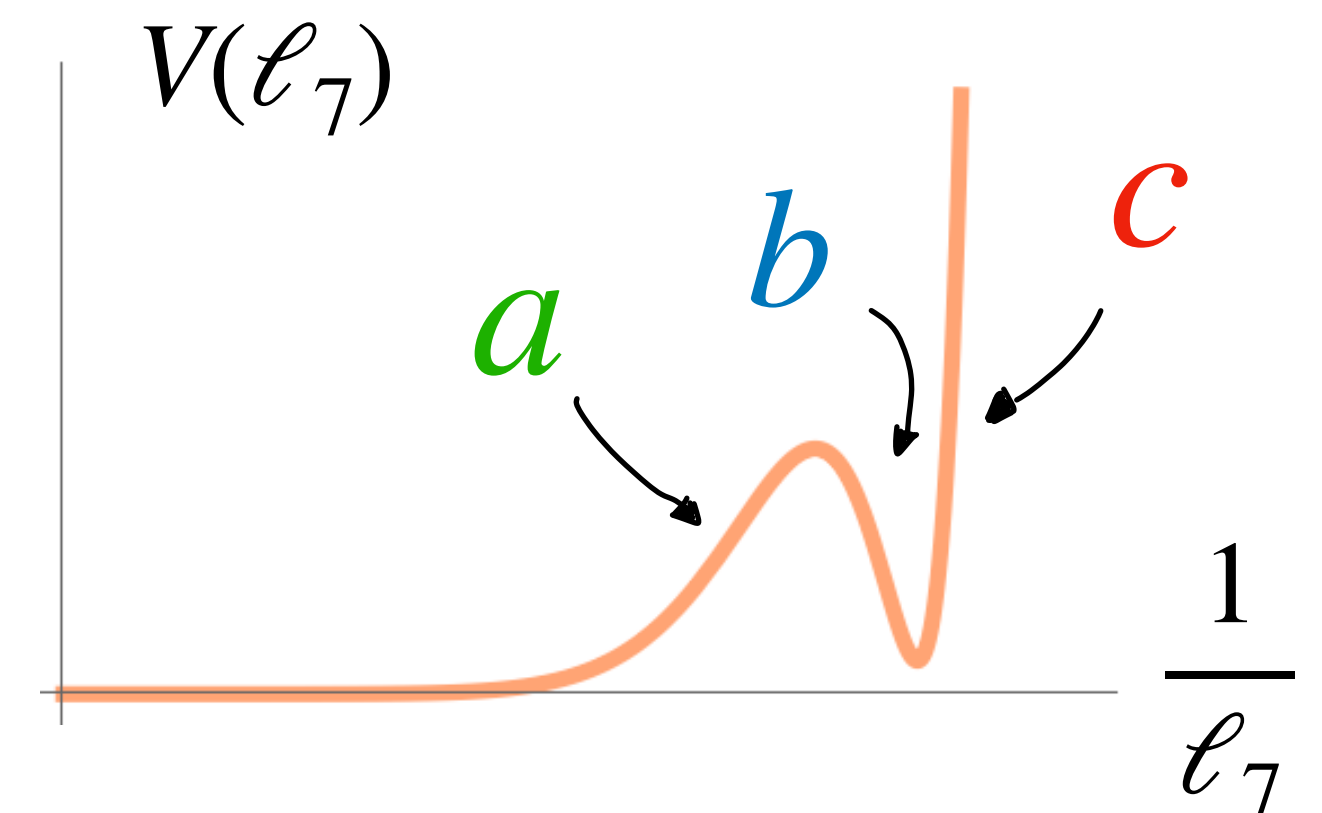
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$$\int \sqrt{g} u^2 \alpha > 0, \text{ and } \int \sqrt{g} u^2 \alpha + \int \sqrt{g} u^2 \gamma \sim - \int \sqrt{g} u^2 \beta$$

Positive first term
Competition of classical and quantum effects

3 terms power law stabilization:



- Stabilization occurs at $\frac{\ell_7}{\ell_{11}} \sim \left(\frac{K}{a}\right)^{1/9} \gg 1$ integrated Casimir

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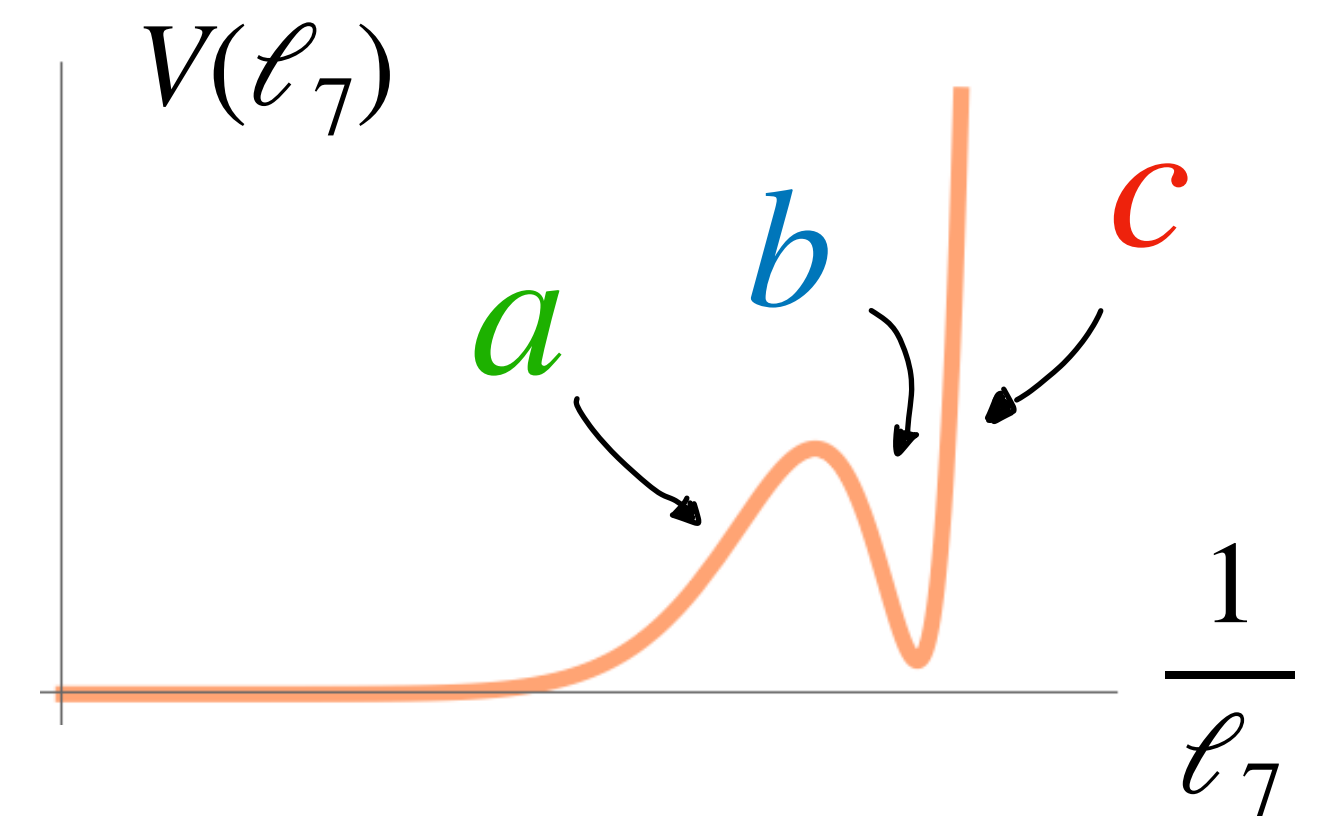
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- To increase a , reduce the flux
- To reduce a , add bulk regions (or reduce cusps)

Backreacted smooth solution in a filled cusp

- At the end of the filled cusp, approximately only radial dependence

[Anderson '06]

- PDEs \rightarrow ODEs

$$ds_{11}^2 = u(y)ds_{4,\Lambda}^2 + dy^2 + R_c^2(y)ds_{\mathbb{T}^5}^2 + R^2(y)d\theta^2$$



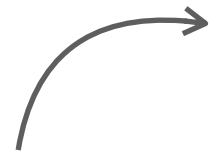
$$0 = 4A' \left(\frac{5R'_c}{R_c} + \frac{R'}{R} \right) + 6(A')^2 - \frac{1}{4}e^{-8A}f_0^2 - \frac{1}{2}e^{-2A}C + \frac{5R'R'_c}{RR_c} - \frac{|\rho_c|}{2R_c^{11}} + \frac{10(R'_c)^2}{R_c^2}$$

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Full set of 11D
EOMs



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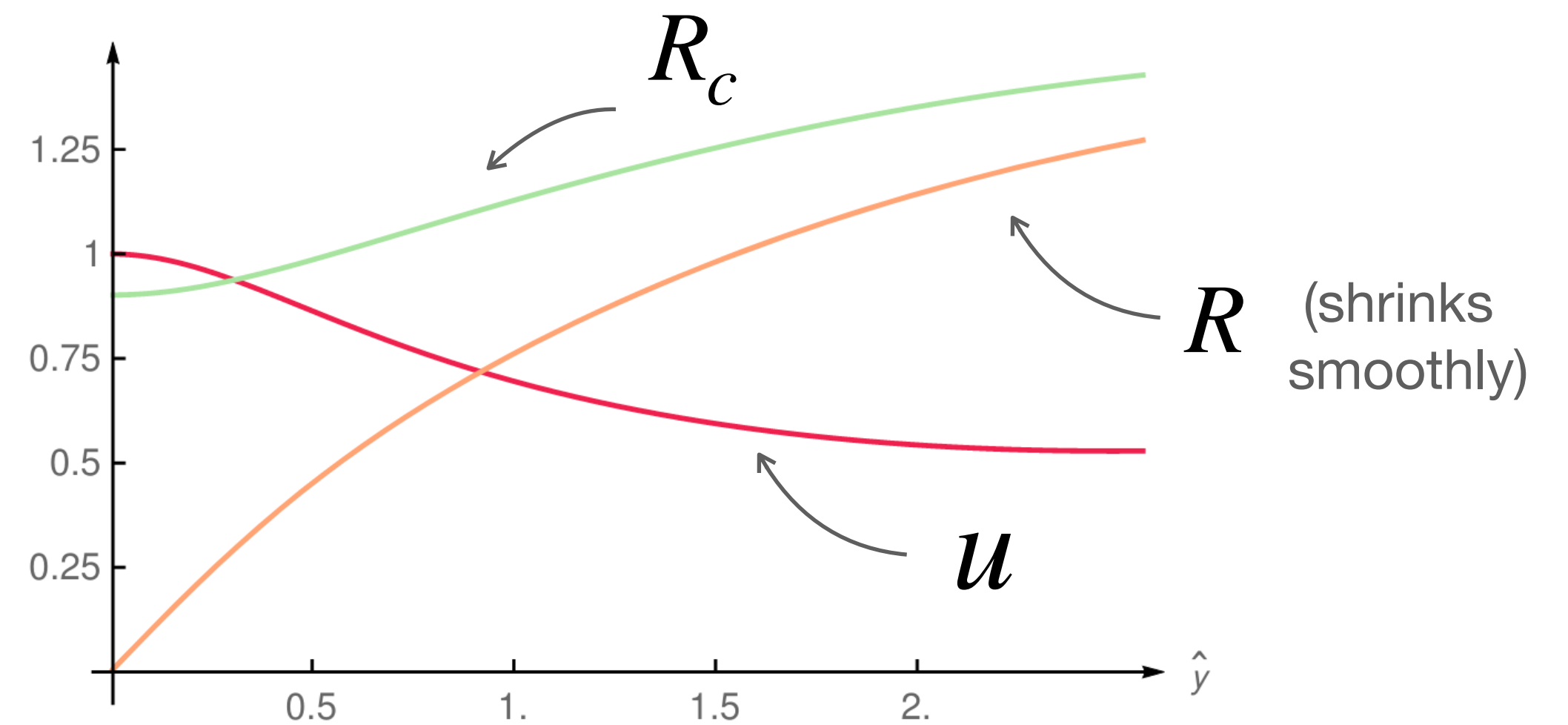
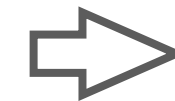
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Full set of 11D
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(functions rescaled for clarity, but can make $R \gg R_c \gg \ell_{11}$)

- Most of the volume is in the cusp

[Italiano, Martelli, Migliorini, '20]

- Gluing to the core of the manifold introduces angular dependence

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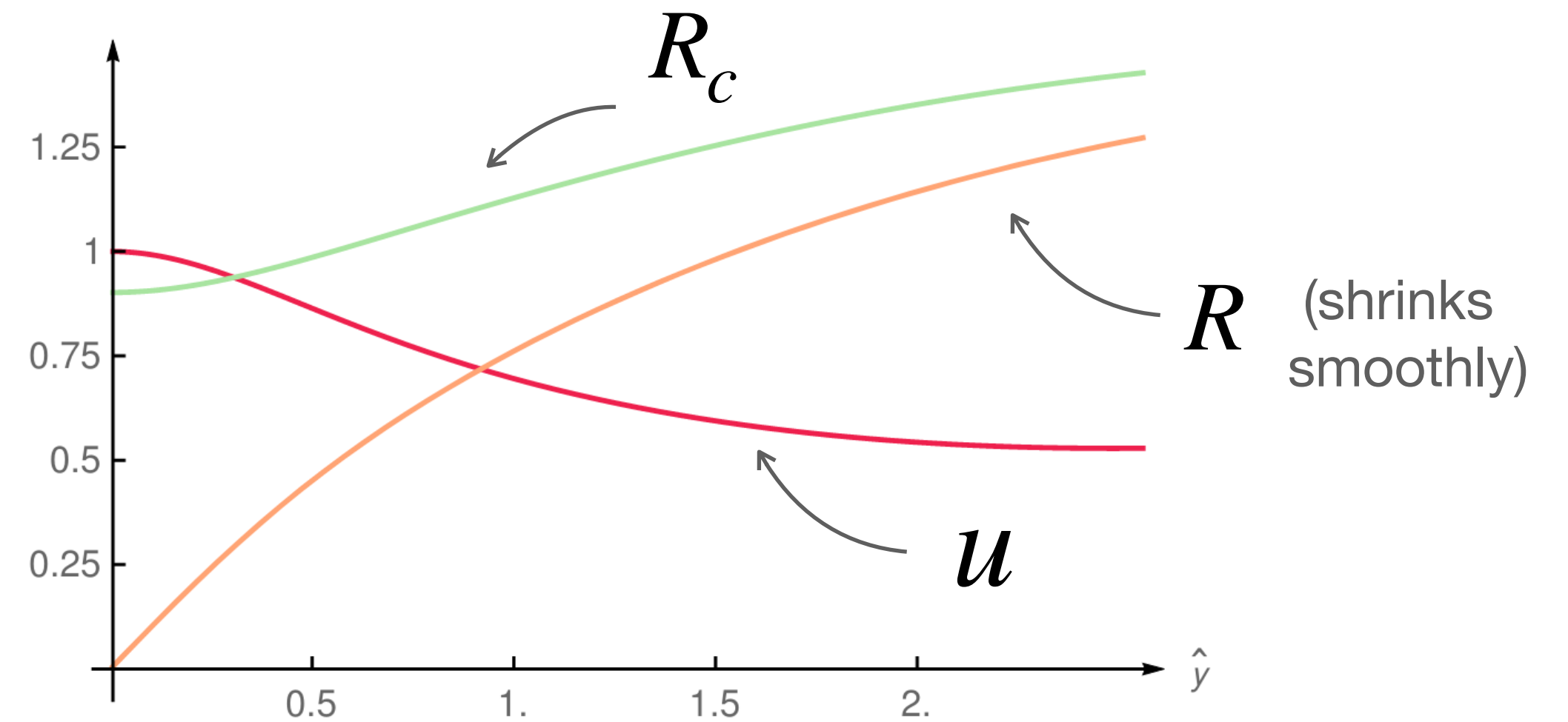
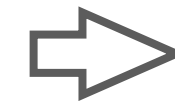
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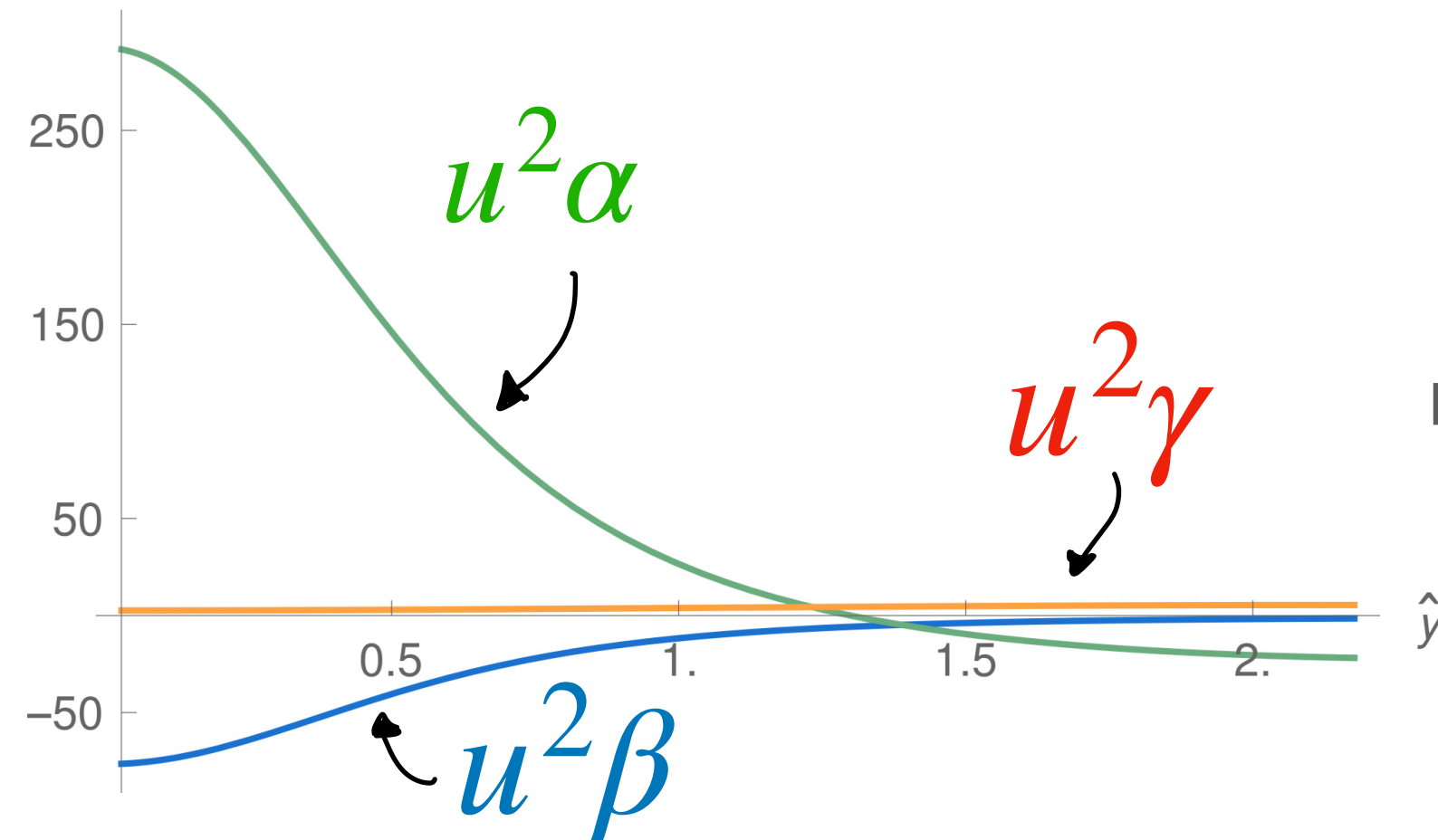
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(functions rescaled for clarity, but can make $R \gg R_c \gg \ell_{11}$)

- We can check our general estimates explicitly:



$$a > 0 \quad \checkmark$$

$$\frac{\int \sqrt{g} u^2 (\alpha + \gamma)}{\int \sqrt{g} u^2 \beta} \sim 1.06 \quad \checkmark$$

$$\delta^2 V > 0 \quad \checkmark$$

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Towards a “fully explicit” solution

- We have described explicit families of hyperbolic manifolds and constructed piece-wise solutions of the equations of motion.
- Compare this with Anderson’s proof of the existence of the filled metric

[Anderson ‘06]

$$\begin{array}{lll} ds_{\text{cusp}}^2 = \frac{dr^2}{r^2} + \frac{r^2}{r_j^2} ds_{T^{n-1}}^2 & \text{Glued at} & ds_{\text{BH}}^2 = \left(\frac{dr^2}{V(r)} + V(r) d\theta^2 + r^2 ds_{\mathbb{R}^{n-2}}^2 \right) / \mathbb{Z}^{n-2} \\ r \geq 0 & r = r_j > 1 & r \geq 1 \quad V(r) = r^2 (1 - r^{1-n}) \end{array}$$

- The gluing is continuous but not smooth, but a nearby smooth Einstein metric is proven to exist

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 - But finding the “fully explicit” metric in this purely geometric setting is numerically challenging and an open problem in hyperbolic geometry, already for $n = 4$ [Martelli ‘15]
 - All cusps need to be filled simultaneously

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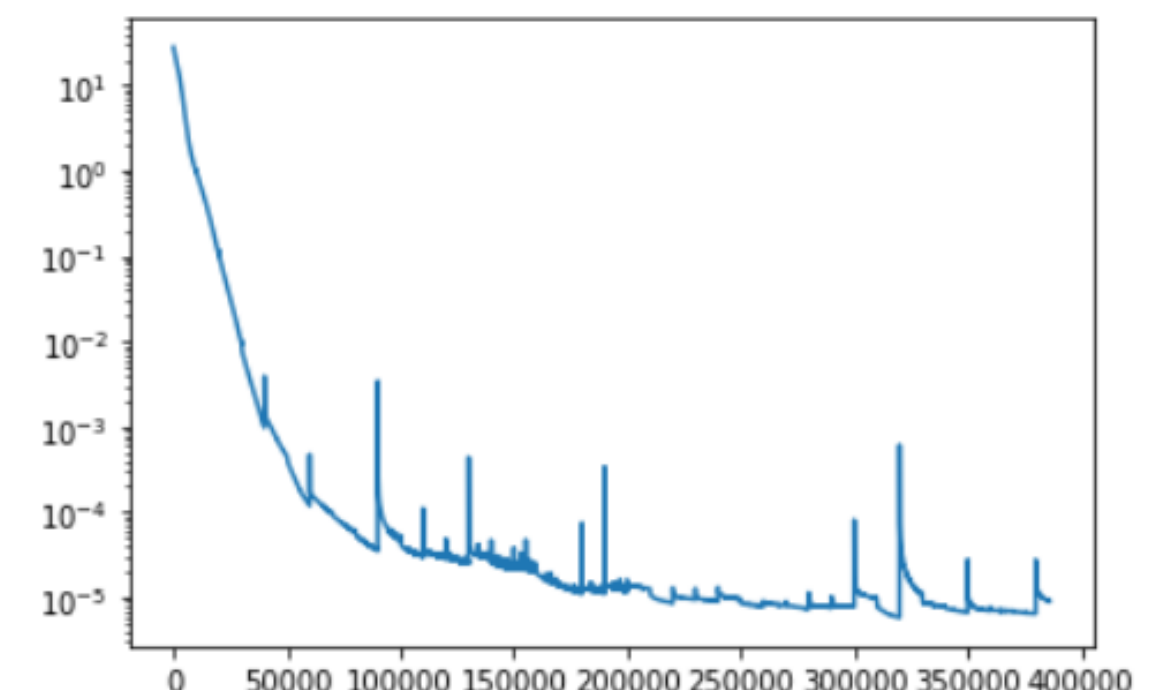
[Martelli ‘15]

- All cusps need to be filled simultaneously

- **In progress:** using Machine Learning techniques to first solve the geometric problem

- Then add Casimir?

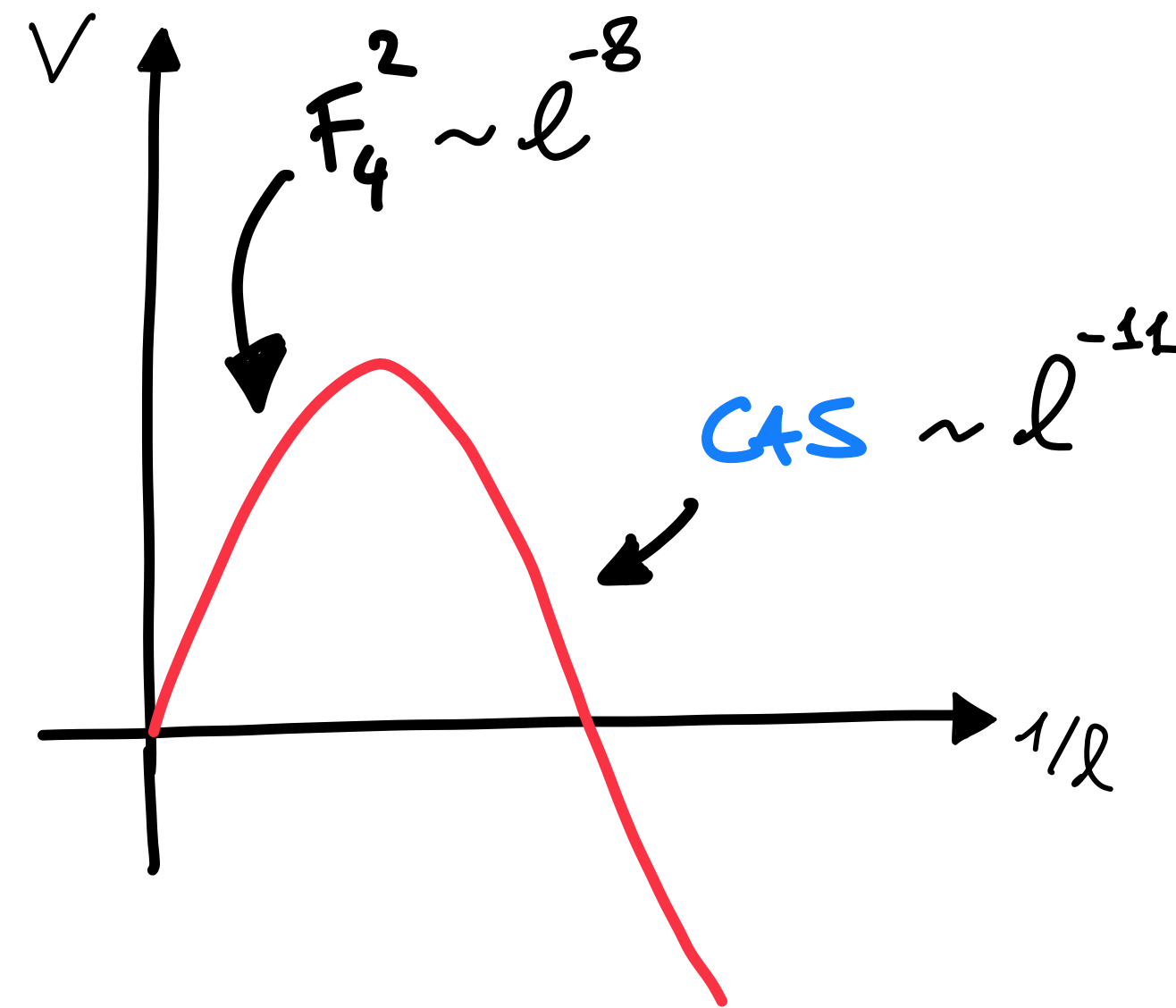
$M = 3$
FILLING 1 CUSP



Thank
you!

An explicit uncontrolled dS with Casimir

- Consider M-theory on $dS_7 \times T^4$ (or $dS_4 \times S^3 \times T^4$), with magnetic F_4 on the torus



$$ds_{11}^2 = L_7^2 ds_{dS_7}^2 + R_c^2 ds_{T^4}^2$$

$$T_{\mu\nu}^{Cas} = |\rho_c| \ell_{11}^9 R_c^{-11} g_{\mu\nu} \quad F_4 = f_4 \text{vol}_{T^4} \quad \Rightarrow \quad \frac{R_c}{\ell_{11}} \sim N_4^{-2/3} \gg 1$$

$$T_{ij}^{Cas} = -\frac{7}{4} |\rho_c| \ell_{11}^9 R_c^{-11} g_{ij} \quad \frac{1}{\ell_{11}^3} \int F_4 = N_4 \quad \Rightarrow \quad \frac{L_7}{\ell_{11}} \sim N_4^{-11/3}$$

Recall for $AdS_4 \times T^7$

$$\left[\frac{R_c}{\ell_{11}} \sim N_7^{2/3} \gg 1 \right]$$

Putting everything together

$$a \equiv \frac{\int_{M_7} \sqrt{g} u^2 \left(-R_7 - 3 \frac{(\nabla u)^2}{u^2} \right)}{\int_{M_7} \sqrt{g} u^2 \frac{42}{\ell_7^2}}$$

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integrated Casimir

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- And:

- Tadpoles around the hyperbolic starting point are bounded and small
- Full Hessian is likely to be positive, gapped:
 - Rigidity + δB stabilized by warp factor

[extending Douglas, '09]

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- Can we obtain it?

Locally (from the EOMs):

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- To reduce a , add bulk regions (or reduce cusps)
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Available tuning
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- Can we also solve all the equations of motion explicitly?